SYNOPSIS FOR PH. D. PROGRAMME

Title of Research:

A study on fuzzy inner product spaces and operator theory in such spaces.

Name of Candidate : Subhendu Ghosal.

Name of the Guide : Dr. Tarapada Bag,
Mathematics Department,
Visva-Bharati, Santiniketan.

Registration No : VB – 1570 OF 2015 – 16

Date of Registration - 28/08/2015

.

Keywords : Fuzzy sesquilinear form, Riesz theorem in fuzzy setting, decomposition theorem, Existence theorem, fuzzy Hilbert adjoint operator, fuzzy self-adjoint operator.

In 1965, Lofti A. Zadeh introduced the idea of fuzzy sets. After that, fuzzy mathematics is developed in different directions. Fuzzy topology, fuzzy algebra, fuzzy con theory, fuzzy dynamical systems etc are the different field of fuzzy mathematics. Fuzzy metric, fuzzy norm and fuzzy inner product are the main tools of fuzzy functional analysis.

In 1984, Osma Kaleva introduced the definition of fuzzy metric space and later on several authors have been working on fixed point theory in fuzzy metric spaces. Kramosil & Michalek introduced an idea of fuzzy metric in different approach. In 1984, Katsaras first introduced the concept of fuzzy norm on a linear space. In 1992 Felbin introduced another concept of fuzzy norm whose corresponding metric is Osma Kaleva type. In 1994, Cheng & Mordeson introduced and idea of fuzzy norm on a linear space in different approach. Following the definition of fuzzy norm introduced by Cheng & Mordeson in 2003, Bag & Samanta redefined the definition of fuzzy norm.

On the other hand study of fuzzy inner product space is relatively recent. Idea of real probabilistic inner product space is introduced by Sklar. Following his concept, Biswas, EI-Abyed & Hamouly first tried to give a meaningful definition of fuzzy inner product space. After that several authors viz. Mazumder & Samanta, Goudarzi & Vaezpour, Hasankhani & Nazari introduced the concept of fuzzy inner product in different approaches.

Our aim is to develop functional analysis in fuzzy setting with an intention to study the properties of fuzzy inner product spaces.

We have a plan of work as in the following :

(i) to study properties of fuzzy inner product spaces introduced by several authors,

(ii) to study operator theory (sesquilinear form, adjoint operator, self -adjoint operator, normal operator, unitary operator, positive operator etc.) in fuzzy inner product spaces.

So far we have made some progress. We establish some results in fuzzy inner product space introduced by Hasankhani & Nazari. We introduce a concept of fuzzy sesquilinear form and establish Riesz theorem for sesquilinear form. Idea of fuzzy adjoint operator, fuzzy self - adjoint operator have been introduced and some properties of them are studied.

We have written the following research papers :

1. S. Ghosal, T. Bag, Fuzzy Sesquilinear Form and its Properties, Journal of Fuzzy Mathematics (communicated).

References:

[1] C. Alsina, B. Schweizer, A. Sklar, On the de_nition of a probabilistic normed space, Aequationes Math 46 (1993) 91-98.

[2] T. Bag, S. K. Samanta, Finite dimensional fuzzy normed linear spaces, The Journal of Fuzzy Mathematics, 11(3) (2003) 687-705.

[3] T. Bag, S. K. Samanta, Fuzzy bounded linear operators, FSS, 151 (2005) 513-547.

[4] T. Bag, S. K. Samanta, Product fuzzy normed linear spaces, The Journal of Fuzzy Mathematics, 13(3) (2005) 545-565.

[5] T. Bag, S. K. Samanta, Fixed point theorems on fuzzy normed linear spaces, Information Sciences, 176 (2006) 2910-2931.

[6] T. Bandyopadhyay, S. K. Samanta, P. Das, Fuzzy metric spaces rede_nd and a _xed point theorem, Bull. Cal. Math. Soc. 81 (1989) 247-252.

[7] R. Biswas, Fuzzy inner product spaces and fuzzy norm function, Information Sciences, 53(1991), 185-190.

[8] S. C. Cheng, J. N. Mordeson, Fuzzy linear operator and fuzzy normed linear spaces, Bull.Cal.Math.Soc. 86 (1994) 429-436.

[9] Zi-ke Deng, Fuzzy pseudo metric spaces, JMAA, 86 (1982) 74-95.

[10] A. M. El-Abyad, H. M. El-Hamouly, Fuzzy inner product spaces, Fuzzy Sets and Systems, 44(2) (1991) 309-326.

[11] C. Felbin, Finite dimensional fuzzy normed linear space, Fuzzy Sets and Systems, 48 (1992) 239-248.

[12] C. Felbin, Finite dimensional normed linear space II, J.Analysis, 7 (1999) 117-131.

[13] M. Goudarzi, S. M. Vaezpour, On the de_nition of fuzzy Hilbert spaces and its
application, The Journal of Nonlinear Science and Applications 2(1) (2009) 46-59.

[14] A.Hasankhani, A.Nazari, M.Saheli, Some properties of fuzzy Hilbert spaces and norm of operators, Iranian Journal of Fuzzy Systems, 7(3) (2010) 129-157.

[15] O. Kaleva, S. Seikala, On fuzzy metric spaces, Fuzzy Sets and Systems 12 (1984), 215-229.

[16] A. K. Katsaras, Fuzzy topological vector spaces II, Fuzzy Sets and Systems 12 (1984) 143-154.

[17] J. K. Kohli, R.Kumar, On fuzzy inner product spaces and fuzzy co-inner product spaces, Fuzzy Sets and Systems 53(2) (1993) 227-232.

[18] I. Kramosil, J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica, 11 (1975) 326-334.

[19] Pinaki Majumdar, S. K. Samanta, On fuzzy inner product spaces, The Journal of Fuzzy Mathematics, 16(2) (2008) 377-392.

[20] Mizumoto, J.Tanaka, Some properties of fuzzy numbers in : M.M.Gupta et al. Editors, Advances in Fuzzy Set Theory and Applications (North-Holland, New-York, (1979) 153-164.

[21] S. Mukherjee, T. Bag, Strictly convex fuzzy normed linear spaces, The Journal of Fuzzy mathematics, 19(2) (2011) 483-494.

[22] S. Mukherjee, T. Bag, Fuzzy real inner product space and its properties, Ann. Fuzzy Math. Inform. 6(2) (2013) 377-389.

[23] S. Mukherjee, T. Bag, Fuzzy inner product space and its properties, International Journal of Fuzzy Mathematics and Systems 5(1) (2015) 57-69.